Symbolic Causality Checking Using Bounded Model Checking

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Abstract. In precursory work we have developed *causality checking*, a fault localization method for concurrent system models relying on the Halpern and Pearl counterfactual model of causation that identifies ordered occurrences of system events as being causal for the violation of non-reachability properties. Our first implementation of causality checking relies on explicit-state model checking. In this paper we propose a symbolic implementation of causality checking based on bounded model checking (BMC) and SAT solving. We show that this BMC-based implementation is efficient for large and complex system models. The technique is evaluated on industrial size models and experimentally compared to the existing explicit state causality checking implementation. BMC-based causality checking turns out to be superior to the explicit state variant in terms of runtime and memory consumption for very large system models.

1 Introduction

In precursory work we have defined a fault localization and debugging technique for concurrent system models called *causality checking* [18,16]. Causality checking relies on *counterfactual reasoning* à la Lewis [21], i.e., an event is considered a cause for some effect in case a) whenever the event presumed be be a cause occurs, the effect occurs as well, and b) when the presumed cause does not occur, the effect will not occur either (counterfactual argument). This simple form of counterfactual reasoning is inadequate to represent logically complex causal structures. In their seminal work [12], Halpern and Pearl have defined a model for causation, based on counterfactual reasoning, that encompasses logically complex relationships amongst events. In our precursory work we have adopted their model and a) related it to models of concurrent computation, in particular transition systems and traces, b) extended it to accommodate the order of events occurring as a causal factor, and c) included the non-occurrence of events as a potential causal factor. The key ingredients of our causality checking algorithm are a complete enumeration of all traces leading into a property violating state, as well as an enumeration of all traces not leading into such a state, in order to establish the counterfactual argument.

An application of causality checking is fault localization within system models. While a model checker will return a simple counterexample for a (non-) reachability property, causality checking will return a temporal logic formula representing the events that are considered to be causal, as well as their order of occurrence in case the order is determined to be causal. The causalities computed by causality checking are much more succinct than counterexamples produced by model checkers and contain more precise error location information than single counterexamples.

We have implemented causality checking up to the work described in this paper most efficiently in the SpinCause tool [20] that relies on explicit state model checking and is based on SpinJa [14], a Java re-implementation of the explicit state model checker SPIN [13]. We have embedded causality checking in our QuantUM tool as the core analysis engine. QuantUM reads system architecture models given in UML or SysML directly out of industrial design tools, such as IBM Rational Rhapsody, performs a reachability analysis for undesired system states using the causality checking components, and outputs the computed causalities as temporal logic fomulae and fault trees [17]. An application of QuantUM is the support of safety cases in the analysis of safety-critical system and software architectures [16,4].

We have applied SpinCause inside the QuantUM context to various industrial sized case studies. At the upper end of the size scale of those case studies the memory consumption of SpinCause starts to be a limiting factor. It is the objective of this paper to propose an implementation of causality checking using an alternative model checking technology, in particular one that relies on bounded model checking (BMC) [6], a symbolic representation of the state space and SAT-solving as a verification engine, in order to evaluate whether this gives us a causality checking implementation which is superior to the explicit state variant in terms of memory consumption.

To this end we define an iterative BMC-based causality checking algorithm. As argued above, in the explicit state causality checking implementation all traces through a system need to be generated. The BMC-based causality checking algorithm presented in this paper uses the underlying SAT-solver invoked by the bounded model checker in order to generate the causal event combinations in an iterative manner. In the course of an iteration only those error traces are generated that contain new information regarding the cause to be computed whereas traces that do not provide new information are automatically excluded from further consideration by constraining the SAT-solver with what is already known about the causal relationships amongst events. With this approach a large number of error traces that would otherwise need to be considered and stored in the explicit state approach can remain unconsidered, which contributes to the memory efficiency of this BMC-based causality checking implementation. We have implemented our algorithm as an addition to the NuSMV2 model checker [9], which encompasses a BMC component, and evaluate its performance using various case studies from various domains and of different sizes. It turns out that for the largest models analyzed the BMC-based implementation requires up to two orders of magnitude less memory than the explicit state implementation. As a consequence, causality checking now scales to a class of significantly more complex models that could previously not be analyzed.

Structure of the Paper. In Section 2 we will present the technical foundations of our work. In Section 3 we describe the proposed iterative BMC-based approach to causality checking. In Section 4 we experimentally evaluate the BMC-based causality checking approach by comparing its performance to the explicit-state causality checking implementation. Related work will be discussed in Section 5 before we conclude in Section 6.

2 Preliminaries

2.1 Running Example

We will illustrate the formal framework that we present in this paper using the running example of a simple railroad crossing system. In this system, a train can approach the crossing (Ta), enter the crossing (Tc), and finally leave the crossing (Tl). Whenever a train is approaching, the gate shall close (Gc) and will open again when the train has left the crossing (Go). It might also be the case that the gate fails (Gf). The car approaches the crossing (Ca) and crosses the crossing if the gate is open (Cc) and finally leaves the crossing (Cl). We are interested in finding those events that are causal for the hazard that the car and the train are in the crossing at the same time.

2.2 System Model

The model of concurrent computation that we use in this paper is that of a transition system:

Definition 1 (Transition System [2]). *A transition system M is a tuple $(S, \mathcal{A}, \rightarrow, I, AP, L)$ where S is a finite set of states, $\mathcal{A}$ is a finite set of actions/events, $\rightarrow \subseteq S \times \mathcal{A} \times S$ is a transition relation, $I \subseteq S$ is the set of initial states, AP is the set of atomic propositions, and $L{:}S \rightarrow 2^{AP}$ is a labeling function.*

Definition 2 (Execution Trace [2]). *An execution trace π in M is defined as an alternating sequence of states $s \in S$ and actions $a \in \mathcal{A}$ ending with a state. $\pi = s_0\ \alpha_1\ s_1\ \alpha_2\ s_2\ \ldots\ \alpha_n\ s_n$, s.t. $s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$ for all $0 \leq i < n$.*

An execution sequence which ends in a property violation is called an error trace or a counterexample. In the railroad crossing example, $s_0 \xrightarrow{\text{Ta}} s_1 \xrightarrow{\text{Gf}} s_2 \xrightarrow{\text{Tc}} s_3 \xrightarrow{\text{Ca}} s_4 \xrightarrow{\text{Cc}} s_5$ is a counterexample, because the train and the car are inside the crossing at the same time.

2.3 Linear Temporal Logic

Linear Temporal Logic (LTL) [22] is a propositional modal logic based on a linear system execution model. An LTL formula can be used to express properties of infinite paths in a given system model.

Definition 3 (Syntax of Linear Temporal Logic). *An LTL formula φ over a set of atomic propositions* AP *is defined according to the following grammar:*

$$\begin{aligned}\varphi ::= \ \text{TRUE} \ \mid \ a \ \mid \ \varphi_1 \wedge \varphi_2 \ \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \Box\varphi \\ \mid \Diamond\varphi \mid \varphi_1 \ \boldsymbol{U} \ \varphi_2\end{aligned}$$

where $a \in$ AP.

The operators $\bigcirc$, $\Box$, $\Diamond$ and $\mathbf{U}$ are used to express temporal behavior, such as *"in the next state sth. happens"*($\bigcirc$), *"eventually sth. happens"*($\Diamond$) and *"sth. is always true"*($\Box$). The **U**-operator denotes the case that *"φ_1 has to be true until φ_2 holds"*. We use $M \models_l \varphi$ to express that an LTL formula φ holds on a system model M and $\pi \models_l \varphi$ for a execution trace in M.

The properties that are expressible in LTL can be separated into two classes, safety and liveness properties. Safety properties can be violated by a finite prefix of an infinite path, while liveness properties can only be violated by an infinite path. For now, causality checking has only been defined for safety properties, namely the non-reachability of an undesired state, which can be characterized using an LTL formula. For instance, the non-reachability property that we want to express in the railroad crossing example is that the train and the car shall never be in the crossing at the same time: $\Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$.

2.4 Event Order Logic

Event Order Logic (EOL) is a linear time temporal logic that is used in causality checking to specify the ordered event occurrences that are computed to be causal. Every EOL formula can be translated into an equivalent standard LTL formula [3].

Definition 4 (Syntax of the Event Order Logic). *Simple event order logic formulae are defined over the set A of event variables:*

$$\phi ::= a \mid \phi_1 \wedge \phi_1 \mid \phi_1 \vee \phi_2 \mid \neg\phi$$

where $a \in A$ and ϕ, ϕ_1 and ϕ_2 are simple EOL formulae. Complex EOL formulae are formed according to the following grammar:

$$\begin{aligned}\psi ::= \phi \mid \psi_1 \wedge \psi_1 \mid \quad & \psi_1 \vee \psi_2 \quad \mid \psi_1 \mathbin{\dot\wedge} \psi_2 \mid \psi_1 \mathbin{\dot\wedge}_{[} \phi \\ \mid \psi_1 \mathbin{\dot\wedge}_{]} \phi \mid & \psi_1 \mathbin{\dot\wedge}_{<} \phi \mathbin{\dot\wedge}_{>} \psi_2\end{aligned}$$

where ϕ is a simple EOL formula and ψ, ψ_1 and ψ_2 are complex EOL formulae.

We define that a transition system M satisfies the EOL formula ψ, written as $M \models_e \psi$, iff $\exists \pi \in M.\ \pi \models_e \psi$. The informal semantics of the operators can be given as follows.

- $\psi_1 \mathbin{\dot\wedge} \psi_2$: ψ_1 has to happen before ψ_2.
- $\psi_1 \mathbin{\dot\wedge}_{[} \phi$: ψ_1 has to happen at some point and afterwards ϕ holds forever.
- $\phi \mathbin{\dot\wedge}_{]} \psi_1$: ϕ has to hold until ψ_1 holds.
- $\psi_1 \mathbin{\dot\wedge}_{<} \phi \mathbin{\dot\wedge}_{>} \psi_2$: ψ_1 has to happen before ψ_2, and ϕ has to hold all the time between ψ_1 and ψ_2.

For example, the formula $\mathrm{Gc} \mathbin{\dot\wedge} \mathrm{Tc}$ states that the gate has to close before the train enters the crossing. The full formal semantics definition for EOL is given in [19].

2.5 Event Order Normal Form

In order to enable the processing of EOL formulas and counterexamples in the BMC-based causality checking algorithm it is necessary to define a normal form for EOL formulas that we refer to as the event order normal form (EONF) [16,3]. EONF permits the unordered *and-* ($\wedge$) and *or*-operator ($\vee$) only to appear in a formula if they are not sub formulas in any ordered operator or if they are sub formulas of the between operators $\dot\wedge_{<}$ and $\dot\wedge_{>}$.

Definition 5. *Event Order Normal Form (EONF) [16,3] The set of EOL formulas over a set $\mathcal{A}$ of event variables in event order normal form (EONF) is given by:*

$$\phi ::= a \mid \neg\phi \qquad\qquad \phi_{\wedge} ::= \phi \mid \neg\phi_{\wedge} \mid \phi_{\wedge_1} \wedge \phi_{\wedge_2}$$

$$\psi ::= \phi \mid \phi_1 \dot\wedge \phi_2 \mid \phi_1 \dot\wedge_{[} \phi_2 \mid \phi_1 \dot\wedge_{]} \phi_2 \mid \phi_1 \dot\wedge_{<} \phi_2 \dot\wedge_{>} \phi_3$$

$$\psi_{\wedge} ::= \psi \mid \phi_{\wedge} \mid \psi_{\wedge_1} \wedge \psi_{\wedge_2} \mid \psi_{\wedge_1} \vee \psi_{\wedge_2}$$

where $a \in \mathcal{A}$ and ϕ are simple EOL formulas only containing single events and $\phi_{\wedge}$, $\phi_{\wedge_1}$, $\phi_{\wedge_2}$ and $\phi_{\wedge_3}$ are EOL formulas only containing the $\wedge$-operator, ψ is a EOL formula containing the ordered operator, and $\psi_{\wedge}$, $\psi_{\wedge_1}$ and $\psi_{\wedge_2}$ are EOL formulas containing the $\wedge$-operator and / or the $\vee$-operator which can be combined with formulas in EONF containing ordered operators.

Every EOL formula can be transformed into an equivalent EOL formula in EONF by rewriting using the equivalence rules defined in [16,3]. For instance, the EOL formula Ta $\dot\wedge$ Gc $\dot\wedge$ Tc can be expressed in EONF as $\psi_{\text{EONF}} = (\text{Ta} \dot\wedge \text{Gc}) \wedge (\text{Gc} \dot\wedge \text{Tc}) \wedge (\text{Ta} \dot\wedge \text{Tc})$.

2.6 Causality Reasoning

Our goal is to identify events that cause a system to reach a property violating state. We hence need to define the notion of causality that we will base our approach on. The notion of causality that we use, as proposed in [15], is based on *counterfactual reasoning* and the notion of *actual cause* defined by Halpern and Pearl in [12]. It not only considers the occurrence of events to potentially be causal, but also the order in which they occur as well as their non-occurrence. For example, an event a may always occur before an event b for an error to happen, but if b occurs first and a afterwards there is no error. In this case, a occurring before b is considered to be causal for the error to happen. Work described in [19] defines when, according to this extended causality notion, an EOL formula ψ describes a causal process for the violation of a non-reachability property, specified using an LTL formula. The causal process [12] consists of the events causing the violation and all events mediating between the causal events and the property violation. Notice that in case there are multiple instances of event occurrences belonging to the same event type in the model, the multiple instances are discriminated. For instance, if along a trace to events of type Gc can be observed, we refer to them as Gc_1 and Gc_2. Otherwise it would not be possible to distinguish between two separate occurrences of the same type of event using standard LTL semantics, which EOL is based on.

Definition 6 (Cause for a property violation [12,18]). *Let π,π' and π'' be paths in a transition system M. The set of event variables is partitioned into sets Z and W. The variables in Z are involved in the causal event chain for a property violation while the variables in W are not. The valuations of the variables along a path π are represented by $val_z(\pi)$ and $val_w(\pi)$, respectively. $\psi_\wedge$ denotes the rewriting of an EOL formula ψ where the ordering operator $\dot\wedge$ is replaced by the normal EOL operator $\wedge$, all other EOL operators are left unchanged. An EOL formula ψ consisting of event variables $X \subseteq Z$ is considered to be a cause for an effect represented by the violation of an LTL property φ, if the following conditions hold:*

- *AC 1: There exists an execution π for which both $\pi \vDash_e \psi$ and $\pi \nvDash_l \varphi$*
- *AC 2.1: $\exists \pi'$ s.t. $\pi' \nvDash_e \psi \wedge (val_x(\pi) \neq val_x(\pi') \vee val_w(\pi) \neq val_w(\pi'))$ and $\pi' \vDash_l \varphi$. In other words, there exists an execution π' where the order and occurrence of events is different from execution π and φ is not violated on π'.*
- *AC 2.2: $\forall \pi''$ with $\pi'' \vDash_e \psi \wedge (val_x(\pi) = val_x(\pi'') \wedge val_w(\pi) \neq val_w(\pi''))$ it holds that $\pi'' \nvDash_l \varphi$ for all subsets of W. In words, for all executions where the events in X have the value defined by $val_x(\pi)$ and the order defined by ψ, the value and order of an arbitrary subset of events on W has no effect on the violation of φ.*
- *AC 3: The set of variables $X \subseteq Z$ is minimal: no subset of X satisfies conditions AC 1 and AC 2.*
- *OC 1: The order of events $X \subseteq Z$ represented by the EOL formula ψ is not causal if the following holds: $\pi \vDash_e \psi$ and $\pi' \nvDash_e \psi$ and $\pi' \nvDash_e \psi_\wedge$*

The EOL formula $\mathrm{Gf} \wedge ((\mathrm{Ta} \wedge (\mathrm{Ca} \,\dot\wedge\, \mathrm{Cc})) \,\dot\wedge_<\, \neg \mathrm{Cl} \,\dot\wedge_>\, \mathrm{Tc})$ is a cause for the occurrence of the hazard in the railroad crossing example since it fulfills all of the above defined conditions (AC 1-3, OC 1) for the corresponding system model that we defined.

2.7 Bounded Model Checking

The basic idea of Bounded Model Checking (BMC) [6] is to find error traces, also called counterexamples, in executions of a given system model where the length of the traces that are analyzed are bounded by some integer k. If no counterexample is found for traces of some length $l \leq k$, then l is increased until either a counterexample is found, or $l = k$. The BMC problem is translated into a propositional satisfiability problem and can be solved using propositional SAT solvers. Modern SAT solvers can handle satisfiability problems in the order of 10^6 variables.

Given a transition system M, an LTL formula f and a bound k, the propositional formula of the system is represented by $[[M, f]]_k$. Let $s_0, ..., s_k$ be a finite sequence of states on a path π. Each s_i represents a state at time step i and consists of an assignment of truth values to the set of state variables. The formula $[[M, f]]_k$ encodes a constraint on $s_0, ..., s_k$ such that $[[M, f]]_k$ is satisfiable iff π is a witness for f. The propositional formula $[[M, f]]_k$ is generated by unrolling the transition relation of the original model M and integrating the LTL property in every step s_i of the unrolling. The generated formula $[[M, f]]_k$ of the whole system is passed to a propositional SAT solver. The SAT solver tries to solve $[[M, f]]_k$. If a solution exists, this solution is considered to be a counterexample of the encoded LTL property.

3 BMC-based Causality Checking

3.1 EOL Matrix

For the BMC-based causality computation with bound k we consider sequences of event occurrences $\pi_e = e_1 e_2 e_3 \dots e_k$ derived from paths of type $\pi = s_0 \xrightarrow{e_1} s_1 \xrightarrow{e_2} s_2 \dots$. We use a matrix in order to represent the fact that certain events occur as well as the ordering of the event occurrences along a trace. This matrix is called EOL matrix.

Definition 7 (EOL matrix). *Let $E = \{e_1, e_2, e_3, \dots, e_k\}$ an event occurrence set and $\pi_e = e_1 e_2 e_3 \dots e_k$ a trace over event occurrences. For integers $i \neq j$ a function o is then defined as follows:*

$$o(e_i, e_j) = \begin{cases} \{\mathsf{TRUE}\} & \text{if } e_i \wedge e_j \\ \varnothing & \text{otherwise} \end{cases}$$

The EOL matrix M_E is constructed from o as follows:

$$M_E = \begin{pmatrix} \varnothing & o(e_1, e_2) & \cdots & o(e_1, e_k) \\ o(e_2, e_1) & \varnothing & \cdots & o(e_2, e_k) \\ \vdots & \vdots & \ddots & \vdots \\ o(e_k, e_1) & o(e_k, e_2) & \cdots & \varnothing \end{pmatrix}$$

where the generated entries in the matrix are either sets of event occurrences or the constant set $\{\mathsf{TRUE}\}$. The empty set $\varnothing$ is also permitted which means no relation for the corresponding event occurrences was found.

Definition 8 (Union of EOL Matrices). *Let M_E, M_{E_1}, M_{E_2} be EOL Matrices with all identical dimensions. The EOL matrix M_E is the union of M_{E_1} and M_{E_2} according to the following rule:*

$$M_{E(i,j)} = M_{E_1(i,j)} \cup M_{E_2(i,j)} \tag{1}$$

for every entry (i, j) in the matrices.

The union of two EOL matrices represents the component-wise disjunction of two matrices. The EOL matrix M_E for an example event sequence in the railroad crossing π = Ca Cc Gf and a refinement EOL Matrix $M'_E = M_E \cup M_{E_{\pi'}}$ using the sequence π' = Gf Ca Cc is created as follows:

$$\begin{array}{l} e_1 = \text{Ca} \\ e_2 = \text{Cc} \\ e_3 = \text{Gf} \end{array} \quad M_E = \begin{pmatrix} \varnothing & \{\mathsf{TRUE}\} & \{\mathsf{TRUE}\} \\ \varnothing & \varnothing & \{\mathsf{TRUE}\} \\ \varnothing & \varnothing & \varnothing \end{pmatrix} \quad M'_E = \begin{pmatrix} \varnothing & \{\mathsf{TRUE}\} & \{\mathsf{TRUE}\} \\ \varnothing & \varnothing & \{\mathsf{TRUE}\} \\ \{\mathsf{TRUE}\} & \{\mathsf{TRUE}\} & \varnothing \end{pmatrix} \tag{2}$$

3.2 EOL Matrix to Propositional Logic Translation

In order to use the information stored in the EOL Matrix in the BMC-based causality checking algorithm a translation from the matrix into propositional logic is needed. First the Matrix is translated into an EOL formula in EONF and afterwards the EOL formula is translated into propositional logic.

Definition 9 (Translation from EOL matrix to EOL formula). *Let M_E a EOL matrix which contains the EOL formula ψ_E and the event set E. $M_{E(i,j)}$ is the set of events in the entry (i,j) in M_E and $e_{(i,j)} \in M_{E(i,j)}$. e_i and e_j denote the ordered events, respectively. Then ψ_E is defined as follows:*

$$\psi_E = \bigwedge_{i=0}^{i=k} \bigwedge_{j=0}^{j=k} \begin{cases} e_i \wedge e_j & \textit{if } e_{(i,j)} = \{\mathsf{TRUE}\} \textit{ and } e_{(j,i)} = \{\mathsf{TRUE}\} \textit{ and } i \neq j \\ e_i \mathbin{\dot\wedge} e_j & \textit{if } e_{(i,j)} = \{\mathsf{TRUE}\} \textit{ and } e_{(j,i)} \neq \{\mathsf{TRUE}\} \textit{ and } i \neq j \end{cases}$$

Lemma 1. *An EOL formula ψ_E obtained via Definition 9 from an EOL matrix M_E is always in Event Order Normal Form (EONF).*

Proof. Sketch: A proof can easily be given using an inductive argument over the rules for the construction of the EOL matrix (Definition 7) and the construction of formula ψ_E (Definition 9).

Using this translation the EOL Matrix from Equation 2 is translated into the following EOL formula in EONF: $\psi_{\text{EONF}} = (\text{Ca} \mathbin{\dot\wedge} \text{Cc}) \wedge (\text{Gf} \wedge \text{Ca}) \wedge (\text{Gf} \wedge \text{Cc})$. The generated EOL formula can be efficiently translated into an equivalent LTL formula as it was shown in [3].

As mentioned in Section 2.3, only safety properties are considered for the BMC-based causality checking approach. Since safety properties can only be violated by finite prefixes of system executions, it is necessary to adapt the definition of a bounded semantics for LTL as defined in [6] for our purposes:

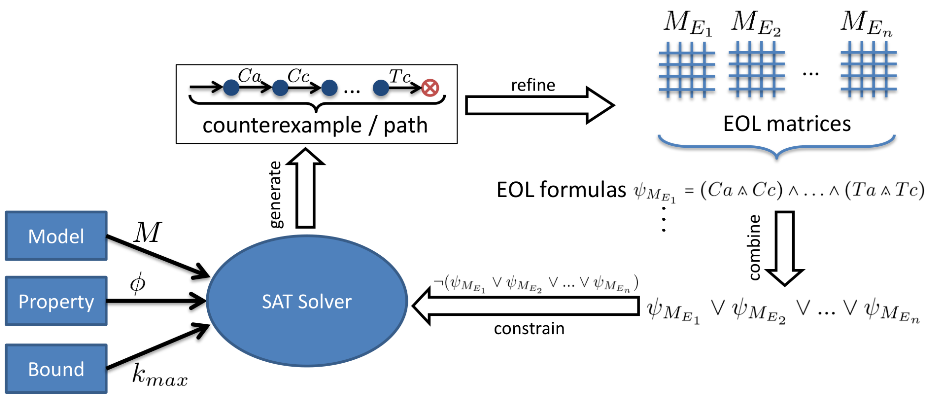
Definition 10 (Bounded Semantics for LTL). *Let $k \geq 0$, and let π be a prefix of an infinite path and $\pi_e = e_0 e_1 e_2 \ldots$ the sequence of events of π. Let ψ_{LTL} an LTL formula obtained by translating an EOL formula ψ into LTL. ψ_{LTL} is valid along π up to bound k, represented by $\pi \vDash_k^0 \psi_{LTL}$, if the following holds:*

$$\begin{array}{lll}
\pi \vDash_k^i p & \textit{iff} & p = e_i \\
\pi \vDash_k^i \neg p & \textit{iff} & p \neq e_i \\
\pi \vDash_k^i f \wedge g & \textit{iff} & \pi \vDash_k^i f \textit{ and } \pi \vDash_k^i g \\
\pi \vDash_k^i f \vee g & \textit{iff} & \pi \vDash_k^i f \textit{ or } \pi \vDash_k^i g \\
\pi \vDash_k^i \Box f & \textit{iff} & \forall j, i \leq j \leq k.\ \pi \vDash_k^j f \\
\pi \vDash_k^i \Diamond f & \textit{iff} & \exists j, i \leq j \leq k.\ \pi \vDash_k^j f \\
\pi \vDash_k^i \bigcirc f & \textit{iff} & i < k \textit{ and } \pi \vDash_k^{i+1} f \\
\pi \vDash_k^i f \boldsymbol{U} g & \textit{iff} & \exists j, i \leq j \leq k.\ \pi \vDash_k^j g \textit{ and } \forall n, i \leq n \leq k.\ \pi \vDash_k^n f
\end{array}$$

The standard translation scheme for translating LTL into propositional logic for a given bound k as described in [6] is used in order to convert the LTL formula ψ_{LTL} into a propositional logic formula.

3.3 The BMC-based Causality Checking Algorithm

According to condition AC 1 it is necessary to know that there exists a counterexample trace which leads to the violation of the considered non-reachability property. In addition, in order to satisfy condition AC 2, however, there need to exist other traces with other events and orderings that do not lead into a violating state. As a consequence, all combinations of events have to be known. In the explicit state causality checking approach [18] all paths through a system need to be computed in order to find all causal events and orderings for a property violation. In order to avoid the explicit computation of all possible paths in the state graph we propose the use of an iterative scheme involving BMC and symbolic constraints on the underlying SAT solver. The symbolic constraint is used in order to find only those paths that contain new information on event orderings and occurrences. This new information is used to strengthen the constraints on the SAT Solver.



Ca
Cc
Tc
counterexample / path
refine
M_{E_1}
M_{E_2}
M_{E_n}
EOL matrices
EOL formulas $\psi_{M_{E_1}} = (Ca \wedge Cc) \wedge \ldots \wedge (Ta \wedge Tc)$
combine
$\psi_{M_{E_1}} \vee \psi_{M_{E_2}} \vee \ldots \vee \psi_{M_{E_n}}$
$\neg(\psi_{M_{E_1}} \vee \psi_{M_{E_2}} \vee \ldots \vee \psi_{M_{E_n}})$
constrain
generate
Model
Property
Bound
M
ϕ
k_{max}
SAT Solver

Fig. 1. The iteration schema of the BMC-based causality checking algorithm

Figure 1 presents the informal iteration scheme of the proposed algorithm. The inputs are the model M, the property ϕ and an upper bound k_{max} for the maximum length of the considered paths. The algorithm starts at level $k = 0$:

Step 1: Generation of Traces. The model M together with the LTL property ϕ and the bound k is converted into a propositional logic formula $[[M, \neg\phi]]_k$. $[[M, \neg\phi]]_k$ is inserted into a SAT solver. The SAT solver tries to find a path that fulfills the given formula. If such a path is found, the algorithm has discovered a counterexample and continues at step 2. Otherwise, the bound k is increased until the first counterexample is found or the maximum bound k_{max} is reached.

Step 2: Matching of EOL Matrices. When a new path π is discovered the set of events E_1 occurring on this trace is compared to the already known EOL matrices, if any. If there is an EOL matrix M_{E_2} covering a set of events E_2 and if $E_1 = E_2$, then the newly discovered orderings of events in E_1 is used to refine the EOL matrix E_2 according to the operation $E_2 := E_2 \cup E_1$ as defined in Definition 8. If there is no matching matrix, a new EOL matrix is created representing a new class of causes [18] containing the ordering of events in π.

Step 3: Combination of new constraints. All EOL matrices M_{E_i} are translated into EOL formulas $\psi_{M_{E_i}}$ according to Definition 9. The translated EOL formulas $\psi_{M_{E_i}}$ are combined disjunctively. In order to exclude the already found orderings from being found again in the next iteration, the result is negated which results in $\varphi' = \neg(\psi_{M_{E_1}} \vee \psi_{M_{E_2}} \vee \ldots \vee \psi_{M_{E_n}})$ with n the number of EOL matrices that have been computed so far.

Step 4: Constraining the SAT Solver. The formula φ' is translated into a propositional logic formula $[[\varphi']]_k$ for a given bound k. $[[\varphi']]_k$ is then used as an additional constraint for the SAT Solver (Definition 10). Afterwards, the algorithm iterates and continues with Step 1.

When the algorithm terminates, the result is stored in the EOL matrices $M_{E_i}, 0 \leq i \leq n$ where n is the number of EOL matrices found during the search.

3.4 Soundness and Completeness

We show that the results generated with the described algorithm are sound up to the pre-defined maximum bound k. Afterwards we will discuss the completeness of the BMC-based causality algorithm.

We first introduce the concept of a candidate set which is a collection of all counterexamples to the considered non-reachability property that have been computed. The elements occurring along the elements of this set are candidates for being causal for the considered property violation.

Definition 11 (Candidate Set (adapted from [19])). *Let n the number of EOL matrices $M_{E_i}, 0 \leq i \leq n$ available at some point during the causality computation, $\neg\phi$ the negation of an LTL reachability property, and $\sum_C$ the set of all counterexamples to the validity of $\neg\phi$ available in the considered system model. The disjunction of all EOL formulas $\psi = \bigvee_{i=0}^{n} \psi_{M_{E_i}}$ generated from the matrices M_{E_i}, is a compact description of all computed counterexamples. The candidate set $\boldsymbol{CS}(\neg\phi) = \{\pi \in \sum_C \mid \forall \pi' \in \sum_C . \pi' \subseteq \pi \Rightarrow \pi' = \pi\}$ contains the minimal set of counterexamples through the system that satisfy ψ.*

Notice that the candidate set is minimal in the sense that removing an event from some trace in the candidate set means that the resulting trace no longer is a counterexample.

Theorem 1. *The candidate set satisfies the conditions AC 1, AC 2.1, AC3 and OC specified in Definition 6.*

Proof. Soundness w.r.t. AC 1: Let $\neg\phi$ the negated LTL property and ψ the EOL formula representing the candidate set $\mathbf{CS}(\neg\phi)$. According to Definition 11, all counterexamples $\pi \in \mathbf{CS}(\neg\phi)$ are traces satisfying $\pi \vDash_l \neg\phi$. $\pi \vDash_e \psi$ holds by the definition of the creation of the EOL Matrices. Therefore AC1 holds for all $\pi \in \mathbf{CS}(\neg\phi)$.

The proofs for the conditions AC 2.1, AC 3 and OC 1 can be constructed in a similar way as shown in [19]. $\square$

What remains to be shown is the soundness with respect to condition AC 2.2, which we shall address next.

Event Non-Occurrence Detection. According to the AC 2.2 test the occurrence of events that are not considered as causal must not prevent the effect from happening. In other words, the non-occurrence of an event can be causal for a property violation. Therefore, we have to search such events and include their non-occurrence in the EOL formulas. In Figure 2 an example is presented which explains this procedure for an EOL formula $\psi = \mathrm{Ca} \wedge \mathrm{Cc} \wedge \mathrm{Ta} \wedge \mathrm{Gc} \wedge \mathrm{Tc}$. Trace

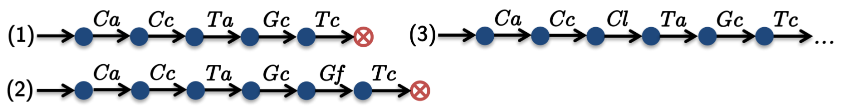


Fig. 2. Three example traces for the EOL-formula $\psi = \mathrm{Ca} \wedge \mathrm{Cc} \wedge \mathrm{Ta} \wedge \mathrm{Gc} \wedge \mathrm{Tc}$. Trace 1 is the minimal trace. While trace 2 (non-minimal) ends in a property violation, trace 3 does not.

1 is the minimal trace ending in a property violation. Trace 2 is non-minimal and also ends in a property violation with the events *Ca, Cc, Ta, Gc, Gf, Tc.* In trace 3 a new event *Cl* appears between *Cc* and *Ta* and no property violation is detected. This means that the appearance of the event has prevented the property violation. In order to transform this appearance into a cause for the hazard, the occurrence is negated and introduced into the EOL formula $\psi = \dots Cc \wedge_< \neg Cl \wedge_> Ta \dots$ The new clause states that *"if between 'the car is on the crossing' and 'the train is approaching the crossing', 'the car does NOT leave the crossing', the hazard does happen"*. In other words: The non-occurrence of *Cl* is causal for the property violation.

A second pass of the algorithm needs to be performed in order to find these non-occurrences. For this second pass the input parameters have to be altered compared to the first pass. The EOL Matrix definition also needs to be extended in order to account for the the possible non-occurrence of events.

Definition 12 (Extended EOL matrix). *Let $E = \{e_1, e_2, e_3, \dots, e_k\}$ an event set and $\pi_e = e_1 e_2 e_3 \dots e_k$ the corresponding sequence. The function o is defined for entries where $i \neq j$ and the function d is defined for entries where $i = j$:*

$$o(e_i, e_j) = \begin{cases} \{\mathsf{TRUE}\} & \text{if } e_i \wedge e_j \\ \phi & \text{if } e_i \wedge_< \phi \wedge_> e_j \\ \varnothing & \text{otherwise} \end{cases} \qquad d(e_i) = \begin{cases} \phi & \text{if } \phi \wedge_] e_i \\ \varnothing & \text{otherwise} \end{cases}$$

The EOL matrix M_E is created as follows:

$$M_E = \begin{pmatrix} d(e_1) & o(e_1, e_2) & \cdots & o(e_1, e_k) \\ o(e_2, e_1) & d(e_2) & \cdots & o(e_2, e_k) \\ \vdots & \vdots & \ddots & \vdots \\ o(e_k, e_1) & o(e_k, e_2) & \cdots & d(e_k) \end{pmatrix}$$

where the generated entries in the matrix are sets of events or the constant set $\{\mathsf{TRUE}\}$. The empty set $\varnothing$ indicates that no relation for the corresponding event configuration was found.

The function o returns *true* if e_1 occurs before e_2 and returns ϕ if e_1 occurs before e_2 and ϕ is true between e_1 and e_2. The function d returns ϕ if ϕ is always occurring before e_i. According to the extended EOL Matrix definition it is possible to insert EOL formulas of the form $e_i \dot{\wedge}_< \phi \dot{\wedge}_> e_j$ and $\phi \dot{\wedge}_] e_i$ into the matrix. This can be used to insert conditions such as $\psi = Cc \dot{\wedge}_< \neg Cl \dot{\wedge}_> Ta$. The special case $e \dot{\wedge}_[\phi$ is not considered here because this will never occur when analyzing safety properties, which is what we focus on in this paper. If a hazard state is reached no future occurrence of any event can prevent the hazard. The formula $e \dot{\wedge}_[\phi$ would encode such a behavior.

Definition 13 (Extended Translation for AC 2.2). *Let M_E an EOL matrix which contains the EOL formula ψ_E and the event set E. $M_{E(i,j)}$ is the set of events in the entry (i,j) in M_E and $e_{(i,j)} \in M_{E(i,j)}$. e_i and e_j denote the ordered events, respectively. Then ψ_E is defined as follows:*

$$\psi_E = \bigwedge_{i=0}^{i=k} \bigwedge_{j=0}^{j=k} \bigwedge_{\substack{\forall e_{(i,j)} \\ \in M_{E(i,j)}}} \begin{cases} e_i \wedge e_j & \textit{if } e_{(i,j)} = \mathsf{TRUE} \textit{ and } e_{(j,i)} = \mathsf{TRUE} \textit{ and } i \neq j \\ e_i \dot{\wedge} e_j & \textit{if } e_{(i,j)} = \mathsf{TRUE} \textit{ and } e_{(j,i)} \neq \mathsf{TRUE} \textit{ and } i \neq j \\ e_i \dot{\wedge}_< \phi \dot{\wedge}_> e_j & \textit{if } \phi = e_{(i,j)} \textit{ and } i \neq j \\ \phi \dot{\wedge}_] e_i & \textit{if } e_{(i,j)} = \phi \textit{ and } i = j \end{cases}$$

The translation from EOL formulas into LTL and further into propositional logic is done according to Definition 10.

Input parameters to the non-occurrence detection. In the second pass of the algorithm, the input parameters for the SAT solver have to be changed. Now the algorithm searches for paths in the system that do not end in a property violation, while fulfilling the EOL formulas that have been found so far. For instance, in Figure 2 trace 3 also fulfills the displayed EOL formula. In order to find those paths the inputs to the SAT solver are the original LTL property ϕ, the original EOL formulas $\psi_{M_{E_i}}$, the model and the bound k. The paths obtained with this method contain the events that prevent the property violation. These events are inserted into a matching EOL matrix. Since the EOL matrices are used to search for the new paths there is always a matching matrix available to the algorithm. The matching of EOL matrices for the AC2.2 condition is defined as follows.

Definition 14 (Matching of paths to EOL Matrix for AC2.2.). *Let π be a path discovered by the second pass, E_π the set of events occurring on π and E_i the event sets of all n EOL matrices. Then the matching EOL matrix is defined according to the following function:*

$$match(\pi) = \{M_{E_i} | \exists i, 0 \leq i \leq n.\ \forall j, 0 \leq j \leq n,\ m_i = max\left(|E_j \cap E_\pi|\right)\}$$

The *match* function returns the EOL matrix M_{E_i} whose event set E_i has the largest number of common events with the event set E_π. Note that there is always a unique maximum for this number: From the definition of the matching of EOL matrices in the first and the second pass of the algorithm two paths containing the same events are merged into one EOL matrix. This means all EOL matrices contain a unique set of events.

The refinement of the matching EOL matrix is conducted according to Definition 8 and 12.

```
Data: $\phi$ the property, $S$ the model, $k_{max}$ the maximum depth of the search
Result: The causal events for a property violation stored in $M_{\text{list}}$
1  $k := 0$;
2  $\psi :=$ FALSE;                                                      /*EOL formula*/
3  $M_{\text{list}} :=$ empty List of EOL matrices;
4  while $k < k_{max}$ do
5      $\pi :=$ solve($\neg\phi$, $S$, $\neg\psi$, $k$);                    /* invoke SAT solver */
6      while $\pi$ is not empty do
7          $m :=$ getMatchingMatrix($M_{\text{list}}$, $\pi$);                        /*Definition 8*/
8          refineEOLMatrix($m$, $\pi$);                                         /*Definition 7*/
9          $\psi :=$ getEOLformula($M_{\text{list}}$);                                /*Definition 9,10*/
10         $\pi :=$ solve($\neg\phi$, $S$, $\neg\psi$, $k$);
11     end
12     $\pi :=$ solve($\phi$, $M$, $\psi$);              /* invoke SAT solver, second pass */
13     while $\pi$ is not empty do
14         $m :=$ getMatchingMatrixAC2_2($M_{\text{list}}$, $\pi$);                  /*Definition 14*/
15         refineEOLMatrixAC2_2($m$, $\pi$);                                    /*Definition 12*/
16         $\psi :=$ getEOLformulaAC2_2($M_{\text{list}}$);                         /*Definition 13,10*/
17         $\pi :=$ solve($\neg\phi$, $S$, $\neg\psi$, $k$);
18     end
19     $k =: k + 1$;
20 end
```

Algorithm 1: BMC-based causality checking algorithm

Theorem 2 (Soundness w.r.t. AC2.2). *For every EOL matrix M_E with the number of events $i = |E|$ the condition AC 2.2 is fulfilled for a maximum number of events x that prevent the property violation from happening and $x = k_{max} - i$.*

Proof. Sketch: Let $\pi \in \mathbf{CS}(\neg\phi)$ be a path of length i in the candidate set of the property violation and $k_{\max}$ the upper bound on the search depth. If $i = k_{\max} - 1$ and there exists a single event that prevents the hazard from happening, the algorithm finds exactly those traces containing this single event and all orderings when processing level $k_{\max}$. If $i = k_{\max} - x$, the same argument applies, and up to x events are found that can prevent the error from happening. □

Completeness. With BMC-based causality checking we can only find event combinations and their orderings up to a predefined bound $k_{\max}$.

Theorem 3. *All EOL matrices discovered with the BMC-based algorithm are complete in terms of conditions AC1,AC2.1, AC2.2, AC3 and OC1 up to the bound k_{max}.*

Proof. Sketch: A proof can be built via structural induction over the generation of the EOL matrices using the minimality argument of the discovered counterexamples.

The completeness of condition AC2.2 is linked to the soundness of this condition and can be proven up to a certain number of events that prevent the property violation from happening. The completeness depends on the number of events in all EOL matrices and the upper bound $k_{\max}$. For example, in Figure 2 trace 3 is at least one step longer than the path resulting in a property violation. This means that if, for example, the maximum bound for the algorithm is set to 5, trace 1 violating the property is found, but trace 3 is not found.

The Algorithm. The pseudo code for the BMC-based causality checking algorithm is presented in Algorithm 1. The function *solve* (Line 5, 10, 12 and 17) converts the input parameters into propositional logic formulas and runs the SAT solver. The result of *solve* is a path of length k satisfying the given constraints.

4 Evaluation

In order to evaluate the proposed approach, we have implemented the BMC-based causality checking algorithm within the symbolic model checker NuSMV2 [9] which also implements BMC. Our CauSeMV extension of NuSMV2 computes the causality relationships for a given NuSMV2 model and an LTL property. The models that we analyze are the Railroad example from Section 2.1, an Airbag Control Unit [1], an Airport Surveillance Radar System (ASR) [4] and a automotive Electronic Control Unit (AECU) that we developed together with an industrial partner. The NuSMV models used in the experiments were automatically synthesized from higher-level design models using the QuantUM tool [17]. The ASR model consists of 3 variants. In the first variant there is only one computation channel for the radar screen (ASR1). In the second and third variant models there are two identical computation channel to raise the availability of the system. In the first two channel variant model the availability of a second channel is modeled by a counter counting component errors (ASR2a), while in the second variant the second channel is a complete copy of the first channel (ASR2b).

All experiments were performed on a PC with an Intel Xeon Processor with 8 Cores (3.60 Ghz) and 144GBs of RAM. We compare our results with the results for the explicit state causality checking approach presented in [18], which were performed on the same computer. For all case studies, a maximum bound of $k = 20$ is chosen. For the considered case studies this value of k is sufficient to compute all relevant causalities. The explicit approach is prallelized using all 8 cores, while the BMC-based approach only uses one core.

In Table 1 the sizes of the different analyzed models are shown. Additionally we compare the number of paths that have to be stored for the explicit causality computation to the iterations needed in the BMC-based setting. For the AECU and the ASR2b the number of traces in the explicit case could not be computed, because the experiments run out of memory.

Figure 3 lists the eol formulas that were computed by the BMC-based causality checking approach. The cause for the occurrence of the considered hazard (a system state in which T_c and C_c hold) is the disjunction of cause 1 and cause 2. Cause 1 represents the case where both the car and the train are approaching the crossing, the car stays on the crossing until the gate closes, and finally the train enters the crossing. Cause 2 represents the case where the gate fails at an

	states	transitions	paths (explicit)	iterations (BMC-based)
Railroad	133	237	47	6
Airbag	155,464	697,081	20,300	24
ASR1	$1 \cdot 10^6$	$7 \cdot 10^6$	$1 \cdot 10^6$	27
ASR2a	$4,6 \cdot 10^7$	$3,3 \cdot 10^8$	$1.5 \cdot 10^7$	32
AECU	$7.5 \cdot 10^7$	$8.6 \cdot 10^8$	-	70
ASR2b	$1 \cdot 10^{12}$	$1 \cdot 10^{13}$	-	208

Table 1. Model sizes in the explicit case and iterations needed for the BMC-based approach.

Cause 1:
$(Ca ⩓ Cc) \wedge (Ca \wedge Ta) \wedge$
$(Ca ⩓_{<} \neg Tl ⩓_{>} Ta) \wedge (Ca ⩓ Gc) \wedge$
$(Ca ⩓ Tc) \wedge (Cc \wedge Ta) \wedge$
$(Cc ⩓_{<} \neg Cl ⩓_{>} Gc) \wedge (Cc ⩓ Gc) \wedge$
$(Ta ⩓ Tc) \wedge (Gc ⩓ Tc) \wedge$
$(Gc ⩓_{<} \neg Cl ⩓_{>} Tc) \wedge (Cc ⩓ Tc) \wedge$
$(Tc ⩓_{<} \neg Tl ⩓_{>} Ca) \wedge (Ta ⩓ Gc)$

Cause 2:
$(Ca \wedge Gf) \wedge (Ca \wedge Ta) \wedge$
$(Ca ⩓_{<} \neg Tl ⩓_{>} Cc) \wedge$
$(Ca \wedge Tc) \wedge (Ca ⩓ Cc) \wedge$
$(Gf \wedge Ta) \wedge (Gf ⩓ Tc) \wedge$
$(Gf \wedge Cc) \wedge (Ta ⩓ Tc) \wedge$
$(Tc ⩓_{<} \neg Tl ⩓_{>} Ca) \wedge$
$(Tc \wedge Cc) \wedge (Cc ⩓_{<} \neg Tl ⩓_{>} tc)$

Fig. 3. Causalities computed for the Railroad Crossing case study.

arbitrary point in time and the car and the train approach and enter the crossing in any possible order. Both causes are consistent with the results obtained by the explicit state causality checking implementation [18] for the same model.

		RT (sec.)	Mem. (MB)
Railroad	explicit	0.73	17.9
	BMC-b.	17.16	121.55
Airbag	explicit	1.61	18.53
	BMC-b.	34.55	192.36
ASR1	explicit	9.24	50.97
	BMC-b.	50.97	303.34
ASR2a	explicit	91.22	826.73
	BMC-b.	186.48	300.54
AECU	explicit	238.13	10,900.00
	BMC-b.	63.0	183.7
ASR2b	explicit	OOM	OOM
	BMC-b.	2,924.74	1,452.45

Table 2. Experimental results comparing the explicit state approach to the BMC-based approach for $k_{max} = 20$. OOM: experiment ran out of available memory.

Discussion Table 2 presents a comparison of the computational resources needed to perform the explicit and the BMC-based causality checking approaches. In order to make the values comparable we limit the search depth for the explicit approach to $k_{max} = 20$ as we have done for the BMC-based approach.

The results illustrate that for the comparatively small railroad crossing model, the airbag model as well as the ASR1 model the explicit state causality checking outperforms the BMC-based approach both in terms of time and memory. For the ASR2 and the AECU models the BMC-based approach uses less memory and finishes the computation faster than in the explicit case. These results reflect a frequently encountered observation when comparing explicit state and symbolic BMC techniques: For small models explicit state model checking is faster and uses less memory since the bounded model checker faces a lot of memory overhead due to the translation of the system into propositional logic. On the other hand, for large models such as ASR2 and AECU the explicit techniques need a lot of memory in order to explicitly store all paths needed to compute the causality classes while the SAT/BMC-based symbolic approach represents whole sets of paths symbolically using propositional logic formulas.

Threats to Validity. The current prototypical tool implementation of the BMC-based causality checking approach, which was used to carry out the experiments described above, is in a somewhat preliminary state. As we argued earlier in the paper, we need to discriminate repeated occurrences of some event type. This requires modifications to the code of the NuSMV, in particular to routines that accomplish the unrolling of the transition relation. The NuSMV code is not designed to be easily modifiable, which is why the proper unrolling accounting for discernible event occurrences of the same type has not yet been fully implemented. As a consequence, the current implementation computes incorrect results for those models for which there are execution paths with repeated occurrences of some event type. However, we believe that this qualitative problem has no significant impact on the quantitative results regarding memory consumption, which are our main concern in this paper. In any event, out of the considered case studies, only the AECU case study contains such events, in all other models this does not happen and the computed causalities are hence correct.

5 Related Work

In [5,10,11] a notion of causality was used to explain the violations of properties in different scenarios. While [5,11] use symbolic techniques for the counterexample computation, they focus on explaining the causal relationships for a single counterexample and thus only give partial information on the causes for a property violation. All of the aforementioned techniques rely on the generation of the counterexamples prior to the causality analysis while our approach computes the necessary counterexamples on-the-fly. Also, our approach is the first and, as far as we know, currently only one that relates the Halpern and Pearl model of causation to the model of transition system and which considers the ordering of events to be potentially causal. In [8] and [7], a symbolic approach to generate Fault Trees [23] is presented. In this approach all single component failures have to be known in advance while in our approach these failures are computed as a result of the algorithm. They do not use an explicitly defined notion of causality, contrary to what we do. The ordering and the non-occurrence of events can not be detected in this approach as being causal for a property violation.

6 Conclusion and Future Work

We have discussed how causal relationships in a system according to the causality checking approach that we previously developed can be established using symbolic system and cause representations together with bounded model checking. The BMC-based causality checking approach presented in this paper was evaluated on six case studies, four of them industrially sized, and compared to the explicit state causality checking approach. It was observed that BMC-based causality checking outperforms explicit state causality checking on large models both in terms of computation time and memory consumption.

In future work the influence of different SAT solving strategies on the speed of discovering new event orderings in the system have to be evaluated. Furthermore, we plan to transform the EOL formulas in EONF into a compact representation in order to enable an automatic Fault Tree generation.

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